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A MODEL OF MULTICOMPONENT GROUTING AND SUFFOSION FILTRATION

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A model of grouting and suffosion filtration of multicomponent systems is proposed. Adequate formulation of the problem and the existence of invariant and self-similar solutions are discussed.

In the filtration of many disperse systems in a porous medium the pores are clogged by disperse particles, which change the filtration characteristics substantially. The main point in studying the filtration of disperse systems in a porous medium, including the effects of clogging (grouting) and emptying (suffosion) of the pores, is creation of mathematical models that reflect adequately the main characteristic features of the process. Investigation of filtration of these systems becomes much more complicated if the disperse phase of the fluid consists of several components whose grouting and suffosion characteristics are different in the same porous medium and under the same hydrodynamic conditions.

A theoretical investigation of filtration of disperse systems with one component of the disperse phase was carried out in [1-6]. A theory of multicomponent filtration, including grouting and suffosion effects, has not been created so far. In the present study on the basis of model concepts used in [7] to create a kinetic model describing changes in the porosity induced by grouting and suffosion, a model of filtration of a multicomponent mixture with account for the difference in the dynamics of sedimentation in pores and escape of the components from the pores is proposed. In order to simplify the mathematics, only grouting and suffosion mechanisms are considered, neglecting other phenomena that change the characteristics of the effective capacity of the medium, such as adsorption of dynamically neutral (or active) particles on the surface of the solid skeleton of the medium, sorption of particles into porous grains composing the porous medium, and development of boundary layers of fluid on the surface of the porous material [8].

Similarly to [7], it is assumed that every pore is a trap for impurity particles and the porous medium is a continuum of traps and each of them can be either vacant or occupied. Pores clogged by impurity particles do not participate in the filtration process, and after they are emptied, the fluid again passes through them. It will be assumed that the particles of each component suspended in the inert phase (usually it is water in the case of filtration of drilling mud, contaminated water, etc.) have different grouting and suffosion activity. If the number of components is m, grouting and suffosion of the pores can be characterized by m pairs of parameters, namely, the probability of capture p_{1i} and release p_{2i} of at least one impurity particle of the i-th component by and from a pore per unit time. To a first approximation the probability of capture can be considered constant, while the probability of release is proportional to the pressure gradient of the liquid flow. The last statement means that release of captured particles from the pores is effected by the hydrodynamic pressure force, while their capture can also take place in the absence of dynamic forces.

As in the case of a monodisperse system, a grouting and suffosion model of a multicomponent liquid flow can be constructed on the basis of a set of equations of material balance, equations describing the kinetics of clogging and emptying of pores, and Darcy's law. The material balance equation for particles suspended in a liquid is written as

$$\frac{\partial n_i}{\partial t} + \operatorname{div}(\mathbf{u} n_i) + \frac{\partial a_i}{\partial t} = \operatorname{div}(D_{\alpha\beta,i}(u)\operatorname{grad} n_i), \quad i = \overline{1, m}, \tag{1}$$

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where n_i is the volumetric concentration of the i-th component in the liquid flow; a_i is the volumetric concentration of captured particles of the i-th component; u is the velocity of the liquid flow; $D_{\alpha\beta,i}(u)$ is the tensor of convective diffusion coefficients of particles of the i-th component; the subscripts α and β refer to the direction of the coordinate axes.

In the grouting and suffosion process, the convective diffusion coefficient $D_{\alpha\beta,i}(u)$ has the same function as in the process of convective diffusion of impurities with or without consideration of sorption of particles when the impurity is assumed to be dynamically neutral. Some authors report that its magnitude near the entrance to the porous medium depends on both the flow velocity and the coordinates [9, 10]. For sufficiently long porous media the dependence of $D_{\alpha\beta,i}(u)$ on the coordinate can be neglected. As was shown in [11], in particular regions of the flow determined by the Reynolds number of the filtration flow, this function is directly proportional to the velocity u, and the proportionality factor for the longitudinal direction is usually an order of magnitude larger than that for the transverse direction.

The equation of grouting kinetics will be taken in the following form:

$$\frac{\partial a_i}{\partial t} = \omega_{1i}(\varepsilon_0 - a_i)\, n_i - a_i\, \omega_{2i}\, P, \quad P = |\operatorname{grad} p|, \quad i = \overline{1, m}, \tag{2}$$

where ω_{1i}, ω_{2i} are parameters characterizing the intensities of sedimentation and escaping of impurity particles of the i-th component (these parameters are determined in terms of p_{1i} and p_{2i}, respectively).

Equation (2) is similar to the equation of the kinetics of porosity for filtration of a monodisperse liquid [7]. The first term in the right-hand side shows that the grouting process can continue up to full clogging of the pores, when the concentration a_i reaches ε_0. Theoretically, only suffosion can proceed in this case. The second term determines the suffosion process.

Since the porosity changes, Darcy's law is taken in the form

$$\mathrm{v}(t) = -K(\varepsilon)\operatorname{grad} p; \tag{3}$$

where $\mathrm{v}(t)$ is the filtration rate.

The function $K(\varepsilon)$ can be taken in various forms. With the assumption of a constant viscosity of the fluidit can be taken that, $K(\varepsilon) = k_0\varepsilon$, $k_0 = \mathrm{const}$ [7]. For a more general case the Karman-Coseny law [12] or other generalized relationships can be used. It should be noted that as mass transfer between the fluid and the porous medium continues, the rheological properties of the former change. This factor can result in a more complicated behavior of $K(\varepsilon)$. However, when emphasis is put on grouting effects, the latter will be neglected.

The velocities of the liquid flow at different points in the medium can be different; meanwhile, the filtration rate can remain constant. This can be explained by changes in the porosity. Assuming that all the localized particles of the components cause clogging of the pores and as a consequence result in changes in the porosity, the local velocity of the flow can be written as

$$u = \frac{v(t)}{\varepsilon} = \frac{v(t)}{\varepsilon_0 - \sum_{i=1}^{m} a_i}. \tag{4}$$

For physical reasons it is clear that

$$\sum_{i=1}^{m} a_i \le \varepsilon_0, \tag{5}$$

where the equality sign can occur theoretically in limiting clogging of the pores by particles, when, according to (2), the grouting process is completed.

For an incompressible liquid the equation of constancy of the total concentration of the components is written as

$$\sum_{i=1}^{m} n_i = n_0 , \quad n_0 = \text{const} . \tag{6}$$

Relations (1) – (6) form a complete set of equations to determine n_i, a_i, ε, $v(t)$, u, and p.

In the one-dimensional case, Eq. (1) has the form

$$\frac{\partial n_i}{\partial t} + \frac{\partial (un_i)}{\partial x} + \frac{\partial a_i}{\partial t} = \frac{\partial}{\partial x}\left(D_i (u) \frac{\partial n_i}{\partial x} \right) , \tag{7}$$

where $D_i(u)$ are the coefficients of longitudinal convective diffusion.

We will consider an adequate formulation of the boundary conditions for Eq. (7) in a porous medium of finite length $[0, l]$. To this end, (7) is multiplied by x^r, $r \geq 0$, and integrated between $(x-\delta)$ and $(x+\delta)$, $\delta \geq 0$:

$$\begin{gathered} \frac{\partial}{\partial t} \int_{x-\delta}^{x+\delta} (n_i + a_i) x^r \, dx + \int_{x-\delta}^{x+\delta} x^r \frac{\partial}{\partial x} \xi_i (t, x) \, dx = 0 , \\ \xi_i (t, x) = un_i - D_i (u) \frac{\partial n_i}{\partial x} . \end{gathered} \tag{8}$$

If $\delta \to 0$, the first integral in (8) tends to zero. From the second integral we obtain

$$x^r \xi_i (t, x) \Big|_{x-\delta}^{x+\delta} - r \int_{x-\delta}^{x+\delta} x^{r-1} \xi_i (t, x) \, dx = 0 . \tag{9}$$

It is easy to see that as $\delta \to 0$ the integral in (9) tends to zero. Consequently, at $r = 0$ and $x = 0$

$$\xi_i (t, x) \Big|_{-\delta}^{\delta} = 0 \quad \text{or} \quad [\xi_i (t, 0)] = 0 , \tag{10}$$

where [ ] is the difference between the values of the function to the right and to the left of a point. Similarly, at $r \geq 0$ and $x = l$ we obtain from (9):

$$l^r \xi_i (t, x) \Big|_{l-\delta}^{l+\delta} = 0 \quad \text{or} \quad [\xi_i (t, l)] = 0 . \tag{11}$$

With (8), (10), and (11), we obtain adequate boundary conditions for (7):

$$v (t) n_i^0 = u^0 n_i (t, 0) - D_i (u^0) \frac{\partial n_i (t, 0)}{\partial x} , \tag{12}$$

$$\left\{ \begin{aligned} & \frac{\partial n_i (t, l-0)}{\partial x} = 0 , \quad D_i (u (t, l+0)) = D_i (u (t, -0)) = 0 , \\ & n_i (t, l-0) = n_i (t, l+0) , \end{aligned} \right. \tag{13}$$

where

$$u^0 = v (t) \Bigg/ \left[\varepsilon_0 - \sum_{i=1}^{m} a_i (t, 0) \right]$$

and n_i^0 are specified initial concentrations of the components in the flow (before entry into the porous medium).

At the asymptotic stage of the process $t \to \infty$, when the concentration gradients n_i at the entrance to the porous medium tend to zero, we obtain from (12)

$$n_i(t,0) = \overset{0}{n}_i \left[\varepsilon_0 - \sum_{i=1}^{m} a_i(t,0) \right]. \tag{14}$$

As can be seen from (12) and (14), the concentration $n_i(t, x)$ at the point $x = 0$ undergoes a discontinuity, although according to (10) the material flow is continuous. It should be also noted that at $x = 0$ the boundary condition contains n_i and a_i simultaneously, because in grouting-suffosion filtration the particles in the flow are not hydrodynamically neutral, as is assumed in problems of convective diffusion or convective transfer, where sorption is included or neglected.

In formulating the problems it should be taken into account that the filtration flow can be simulated by specifying either $v(t)$ or the pressure p at two points of a finite porous medium.

At the asymptotic stage $t \to \infty$ the process becomes quasi-equilibrium, when the condition $\partial a_i/\partial t = 0$ can be assumed in kinetic equations (2). Then, steady-state values of a_i are determined as

$$(a_i)_{st} = \frac{\omega_{1i}\,\varepsilon_0\,\overset{0}{n}_i}{\omega_{1i}\,\overset{0}{n}_i + \omega_{2i}\,P}, \quad \overset{0}{n}_i = n_i(\infty, x) = \text{const}, \quad i = \overline{1, m}. \tag{15}$$

Let the filtration flow be simulated by specifying $v(t) = v_0 = \text{const}$. The kinetic process of grouting and suffosion is assumed to be quasi-equilibrium. Then, with (3) in the one-dimensional case, we obtain from (2):

$$\omega_{1i}(\varepsilon_0 - a_i)\,n_i - a_i\,\omega_{2i}\,v_0/K \left(\varepsilon_0 - \sum_{i=1}^{m} a_i \right) = 0, \quad i = \overline{1, m}. \tag{16}$$

Equations (16) form a nonlinear algebraic set. Its solutions are assumed to be $a_i = f_i(n)$, where $n = (n_i, n_2, \ldots, n_m)$. Then with $D_i(u) = 0$ we have from (7):

$$\frac{\partial (n_i + f_i(n))}{\partial t} + \frac{\partial}{\partial x}(u n_i) = 0, \quad i = 1, m. \tag{17}$$

Summing (7) over all i and considering (6), we obtain

$$\frac{\partial R}{\partial t} + n_0 \frac{\partial u}{\partial x} = 0, \quad R = \sum_{i=1}^{m} f_i(n). \tag{18}$$

Equations (17) and (18) form a system of quasilinear equations. In (17) $m-1$ equations are independent because relation (6) relates all m solutions n_i. In matrix form the system is written as

$$A \frac{\partial v_k}{\partial t} + uE \frac{\partial v_k}{\partial x} = 0, \quad i, k = \overline{1, m}, \tag{19}$$

where

$$A = \{ a_{ik} \}; \quad E = \{ e_{ik} \};$$

$$n_m = n_0 - \sum_{i=1}^{m-1} n_i; \quad v_k = \begin{pmatrix} n_1 \\ n_2 \\ \cdot \\ \cdot \\ \cdot \\ n_{m-1} \\ u \end{pmatrix};$$

$$a_{ii} = 1 + f_{ii} - f_{im} - \frac{n_i}{n_0} \sum_{j=1}^{m} (f_{ji} - f_{jm}) ;$$

$$a_{mm} = a_{im} = 0 ; \quad e_{ik} = 0 , \quad e_{ii} = 1 ;$$

$$a_{ik} = f_{ik} - f_{km} - \frac{n_i}{n_0} \sum_{j=1}^{m} (f_{jk} - f_{jm}) ;$$

$$a_{mi} = \frac{u}{n_0} \sum_{j=1}^{m} (f_{ji} - f_{jm}) ; \quad f_{ii} = \frac{\partial f_i}{\partial n_i} ; \quad f_{ik} = \frac{\partial f_i}{\partial n_k} .$$

Systems of equations (17)–(19) coincide formally with the system of equations for sorption of multicomponent mixtures in a granular porous medium with an infinitely large mass-transfer coefficient within a porous grain [13-15]. However, this similarity is only superficial. Quasi-equilibrium conditions in sorption problems are determined completely by the sorption is isotherms. In the present grouting and suffosion model of filtration these conditions depend on hydrodynamic parameters of the flow. Moreover, without the assumption of dynamic neutrality in grouting filtration, the behavior of the process changes substantially. Nevertheless, the general analysis of system (17), (18) or (19) can be quite similar to that of the system of equations for sorption [13-15].

The eigenvalues μ_i, $i = \overline{1, m}$, of the matrix A are determined from the equation

$$\det (A - \mu E) = 0 . \tag{20}$$

Since $a_{am} = 0$, $i = \overline{1, m}$, and consequently, the rank of the matrix is $m-1$, for positive μ_i they can be ordered:

$$\begin{gathered} 0 = \mu_m < \mu_{m-1} < \ldots < \mu_2 < \mu_1 , \\ 0 < \lambda_1 < \lambda_2 < \ldots < \lambda_{m-1} , \end{gathered} \tag{21}$$

where $\lambda_i = u/\mu_{m-i}$, $i = \overline{1, m-1}$.

Grouting of the pores by individual components necessitates satisfaction of conditions (21). In this case the system of equations (19) is called hyperbolic in the narrow meaning of the word [16].

In the general case system (17) and (18) should be solved numerically, but under certain conditions, when the characteristics of the problem are functions of the variable, invariant solutions of the type of running waves

$$z = x - wt , \tag{22}$$

or similarity solutions of the type of divergent waves

$$\eta = x/t . \tag{23}$$

are possible.

Let the porous medium have the following distribution of concentrations at the initial moment:

$$n_i (0 , x) = n_{0i} , \quad a_i (0 , x) = 0 , \tag{24}$$

where n_{0i} = const, and on the boundary $x = 0$

$$n_i (t , 0) = n_i^0 , \quad n_i^0 = \text{const} . \tag{25}$$

is simulated.

The problems of existence, uniqueness, and single-valuedness of running wave solutions (22) of system (19) are studied similarly to [13, 14].

For running wave solutions (22), boundary conditions (24) and (25) take the form

$$n_i(\infty) = n_i^{(s)}, \quad n_i(-\infty) = n_i^{(s+1)},$$
$$u(\infty) = u^{(s)}, \quad u(-\infty) = u^{(s+1)},$$
$$a_i(\infty) = a_i^{(s)} = 0, \quad a_i(-\infty) = a_i^{(s+1)} = (a_i)_{\rm st},$$
$$dn_i(\pm\infty)/dz = 0, \quad da_i(\pm\infty)/dz = 0, \quad i = \overline{1, m}, \tag{26}$$

$$z = x - w_i^{(s)} t, \tag{27}$$

where the values of $n_i^{(s)}$ and $n_i^{(s+1)}$ lie in the interval $[n_{0i}, n_i^0]$ and $w_i^{(s)}$ is the velocity of propagation of concentration waves for the i-th component.

With (26) in view and using the variable z from (27), we obtain from (7) and (2):

$$D_i(u)\frac{dn_i}{dt} = un_i - u^{(s)} n_i^{(s)} - w_i^{(s)}(n_i - n_i^{(s)} + a_i) = L_{1i}(v),$$
$$-w_i^{(s)}\frac{da_i}{dz} = \omega_{1i}(\varepsilon_0 - a_i)n_i - a_i\omega_{2i}P = L_{2i}(v), \tag{28}$$

where $v_j = (n_j, a_j, u)$, $j = \overline{1, m}$.

According to (26), the points $v^{(s)}$ and $u^{(s+1)}$ are zeros of the operators $L_{1i}(v)$ and $L_{2i}(v)$, i.e.,

$$L_{1i}(v^{(s)}) = L_{1i}(v^{(s+1)}) = L_{2i}(v^{(s)}) = L_{2i}(v^{(s+1)}) = 0.$$

For the running wave solution, the condition for existence of the integral curve connecting the points $v^{(s)}$ and $v^{(s+1)}$ can be found on the basis of considerations similar to those in [13, 14]. For the problem under consideration it has the form

$$\frac{u^{(s+1)}}{w_i^{(s)}} + \frac{n_i}{n_0}\frac{R - R^{(s+1)}}{n_i - n_i^{(s+1)}} - \frac{a_i - a_i^{(s+1)}}{n_i - n_i^{(s+1)}} > 1 > \frac{u^{(s)}}{w_i^{(s)}} + \frac{n_i}{n_0}\frac{R - R^{(s)}}{n_i - n_i^{(s)}} - \frac{a_i - a_i^{(s)}}{n_i - n_i^{(s)}}. \tag{29}$$

Similarly, it can be shown that

$$\lambda_i(v^{(s+1)}) > w_i^{(s)} > \lambda_i(v^{(s)}), \tag{30}$$

is the condition for existence of a single curve connecting the points $v^{(s)}$ and $v^{(s+1)}$, and

$$\lambda_{i-1}(v^{(s+1)}) < w_i^{(s)} < \lambda_{i+1}(v^{(s)}). \tag{31}$$

is the condition for single-valuedness of the integral curve for monotonic functions f_{ii}, f_{ik}.

When condition (31) is satisfied, running waves of concentration for various components, both in n_i and in a_i, are behind or ahead of one another.

Thus, conditions (29)-(31) ensure existence, single-valuedness, and uniqueness of solutions of the type of running waves of concentration.

NOTATION

A, matrix of the coefficients in system (19); a_i, volume concentrations of particles clogging pores; $D_{\alpha\beta i}$, tensor of convective diffusion coefficients of particles; $D_i(u)$, longitudinal convective diffusion coefficient; E, unit matrix in (19); $K(\varepsilon)$, permeability coefficient; l, length of the model of the porous medium; n_0, n_i, total and component concentrations of particles in the flow; n_{0i}, n_i^0, initial and boundary values of concentrations of particles; p, pressure; t, time, u, physical velocity of liquid motion; $v(t)$, filtration rate; w, velocity of concentration waves; x, coordinate; z, invariant variable; δ, positive constant in (8); ε_0, ε, initial and instantaneous porosities; $\eta = x/t$, self-similar variable; μ_i, eigenvalues of the matrix A; ω_{1i}, ω_{2i}, parameters of intensities of sedimentation and escape of particles from pores. Subscript i refers to the i-th component of the mixture.

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